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PARTICLE DRAG IN A DILUTE TURBULENT TWO-PHASE SUSPENSION FLOW

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Abstract—The drag between phases plays an important role in the study of a turbulent two-phase suspension flow and its physical understanding will greatly promote progress in theoretical treatments of a whole range of important industrial and technical problems involving such a flow. The conventional practice of using the results of measurements based on a single particle in a laminar stream for the case of a turbulent flow of a dilute suspension is questioned. An analysis of the results of local measurements of upward turbulent flows of a solid particle–air two-phase suspension leads to the determination of the realistic particle drag coefficient over a wide range of flow conditions. It is established that the particle drag can be described by the simple Stokes law, based on an apparent turbulent viscosity of the fluid for the particles in the suspension flow. A correlation is provided for this apparent turbulent viscosity in terms of the particle size and concentration in the suspension, the local flow turbulence Reynolds number and the particle-to-fluid density ratio.

INTRODUCTION

The interaction forces between the phases play a pivotal role in the motion of a two-phase suspension flow and their physical understanding will contribute significantly to the advancement of the theoretical treatment of such important industrial problems as deposition of particles on walls, particle separation, droplet behavior in sprays etc. In general, these forces depend on the local flow characteristics as well as the interactions between particles. For small particle Reynolds numbers of suspension flows of sufficient diluteness, the Stokes drag law, which is based on an unbounded laminar stream passing over a single spherical particle, has been generally regarded as an acceptable approximation for laminar as well as turbulent two-phase suspension flows:

$$C_D = \frac{24}{\mathrm{Re}_p} \quad (\mathrm{Re}_p \ll 1), \qquad [1]$$

where

$C_D = \dfrac{P_d}{(\frac{1}{2}\rho_f U_r^2)(\frac{1}{4}\pi d_p^2)}$, the drag coefficient,

F_D = the drag force on the particle,

ρ_f = the fluid density,

$U_r = U_0 - U_p$, the fluid-to-particle relative velocity,

U_0 = the time-mean fluid velocity,

U_p = the time-mean particle velocity,

d_p = the particle diameter,

$\mathrm{Re}_p = d_p U_r/\nu_f$, the particle Reynolds number,

$\nu_f = \mu_f/\rho_f$, the fluid kinematic viscosity

and

μ_f = the fluid dynamic viscosity.

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For slightly larger particle Reynolds numbers, approximations to the early portion of the standard drag curve have been used, which expresses the drag coefficient C_D as a function of particle Reynolds number Re_p, e.g. Durst *et al.* (1984).

However, the experimental data for the correlation which establishes the standard drag curve $C_D(Re_p)$ have been obtained by measuring the drag coefficient of a single sphere in a uniform laminar flow. It is therefore highly questionable for such results to be applicable to the case of a turbulent flow of a two-phase suspension in which the turbulence inherent in the flow and its modification brought about by the dynamic interaction between the phases are expected to play an important role. One of the main conceptional difficulties lies in the fact that, usually, in applying this kind of correlation the inappropriate molecular viscosity of the fluid has been incorporated. The more appropriate apparent turbulent viscosity of the fluid for a particle in a suspension flow remains unknown and a direct measurement of the drag force on a particle is still extremely difficult for such a complex flow as the turbulent flow of a two-phase suspension.

There have been attempts to directly measure the drag force on a particle from particles maintained in a statistically neutral state of suspension in an upward turbulent two-phase suspension flow in a vertical pipe, e.g. Lewis *et al.* (1949), Harada *et al.* (1984), Siegel (1970), Flatow (1973) and Sunami *et al.* (1978). From a knowledge of the fluid velocity and the weight of the particle, a drag coefficient C_D can be deduced. It is considerably different from that for a single particle in a uniform laminar flow. Since there are significant variations in the flow properties across the pipe radius, results from such gross observations cannot be taken quantitatively seriously. However, the qualitative indication of the apparent deviation of the particle drag coefficient from the standard drag curve is of great significance.

There have also been attempts to indirectly measure the drag force on a particle from particle arrays fixed in space by mechanical supports in a turbulent pipe flow by measuring the pressure gradient along the pipe, e.g. Stinzing (1971). By subtracting the pressure gradient for the corresponding single-phase flow without the presence of particles and the minor contribution from the mechanical supports from the measured pressure gradient, one can deduce a pressure gradient which is supposedly caused by the two-phase flow interaction. A drag coefficient C_D calculated from this deduced pressure gradient has been found to be significantly smaller than that for a single particle in a uniform laminar flow. The most serious drawback of this kind of approach is the unjustifiable assumption of the pipe resistance for a two-phase mixture to be the same as that for a single-phase flow without the presence of particles. Additional shortcomings include the lack of free lateral movement of the particles as would be found in real suspension flows and the practical limitation of the scheme to relatively large particles.

PARTICLE DRAG FROM LOCAL MEASUREMENTS

Fairly recently two reports appeared in the literature on the local measurements of flow properties of an upward solid particle–air two-phase turbulent suspension flow in a vertical pipe by the use of laser–Doppler anemometry.

Lee & Durst (1982) described experiments with the flow of air carrying in suspension single-sized glass spheres of 100, 200, 400 or 800 μm in a vertical pipe of 41.8 mm dia. The anemometer was provided with amplitude discrimination and frequency filter banks which together were used to differentiate between fluid and particle velocities. Signals were captured on a transient recorder and transferred to a computer for processing. Mean velocity profiles were measured both for the air and the particles.

Tsuji *et al.* (1984) described experiments with the flow of air carrying in suspension single-sized plastic spheres of 200 μm, 500 μm or 3 mm in a vertical pipe of 30.5 mm dia. A frequency tracker converted the frequency of a Doppler burst signal into a voltage output proportional to the velocity. The Doppler signal was obtained not only from the small tracers but also from the spherical test particles. The separation of signals of the test particles from those of the tracers was made possible by making use of a specially designed signal-discrimination device. The principle of the signal discriminator is that the burst signals with sufficiently large pedestal components come from the large particles, while those with small pedestal but large Doppler components come from the small tracers that follow the fluid motion. Mean velocity profiles were measured both for the

Table 1. Characteristics of the solid particle–air two-phase suspension flows used in local measurements

	Particle-to-air density ratio, $S = \rho_s/\rho_f$	Fow Reynolds number $Re \times 10^{-4}$	Particle diameter, d_p (m)	Particle-to-air mass flux ratio, ratio, m	Centerline velocity, U_0 (m/s)	Centerline particle velocity, U_p (m/s)	Centerline relative velocity, $U_r = U_0 - U_p$ (m/s)
Lee & Durst (1982)	1833 (glass spheres)	1.21	0.0001	1.06	5.70	5.17	0.53
			0.0002	1.15	5.84	4.16	1.68
			0.0004	1.32	5.77	2.28	3.49
			0.0008	2.22	5.66	1.43	4.23
Tsuji *et al.* (1984)	833 (plastic spheres)	3.1	0.0002	4.2	14.60	13.14	1.46
				2.1	17.40	13.92	3.48
				1.0	18.90	15.59	3.31
		1.6	0.0005	3.6	8.07	4.80	3.27
				2.0	9.00	4.46	4.54
				1.1	9.65	3.83	5.82
		3.0	0.003	3.0	19.50	8.90	10.60
				2.2	19.50	8.40	11.10
				1.2	20.00	8.20	11.80

air and the particles. Table 1 shows the characteristics of solid particle–air two-phase suspension flows used in local measurements of these two reports.

The particle-to-air mass flux ratio m used in these experiments can be readily identified with the flow situation along the centerline. A force balance between gravity and drag on a single particle from among a suspension of particles along the centerline yields the particle drag coefficient:

$$C_D = \frac{(\frac{1}{6}\pi d_p^3)\,\rho_p g}{(\frac{1}{4}\pi d_p^2)\,(\frac{1}{2}\rho_f U_r^2)} = \frac{4}{3}\frac{S d_p g}{U_r^2}, \qquad [2]$$

where

$$S = \frac{\rho_p}{\rho_f}, \text{ the particle-to-air density ratio}$$

and

$$g = \text{gravitational acceleration.}$$

A plot of the particle drag coefficient C_D against the particle Reynolds number from these experiments is shown in figure 1 where the classical standard drag curve for a single sphere in a laminar stream is also plotted for comparison. It is apparent from the plot that the drag coefficient for a particle in a suspension is always significantly lower than that for a single particle in a laminar stream, mostly by an order of magnitude.

ANALYSIS

An apparent contribution to the difference between the drag coefficient for a particle in a two-phase suspension and that for a single particle in a uniform flow is the presence of other particles in the flow which occupy a finite amount of volume in the mixture. For a laminar two-phase suspension flow, Tam (1969), Zuber (1964) and Murray (1967) collectively proposed a drag coefficient which is obtained from that for a single particle in a uniform flow modified through the effect of the particle volumetric concentration α. In the same spirit, Ishii & Zuber (1979) proposed that in the viscous regime the drag coefficient C_D for a turbulent two-phase suspension flow has exactly the same functional form in terms of modified particle Reynolds number as the drag coefficient for a single particle in a uniform flow in terms of the particle Reynolds number, the standard drag curve. The modified Reynolds number is based on a modified viscosity μ_m of the fluid due to the presence of other particles in the mixture, as proposed by Ishii (1977), in terms of the present notation:

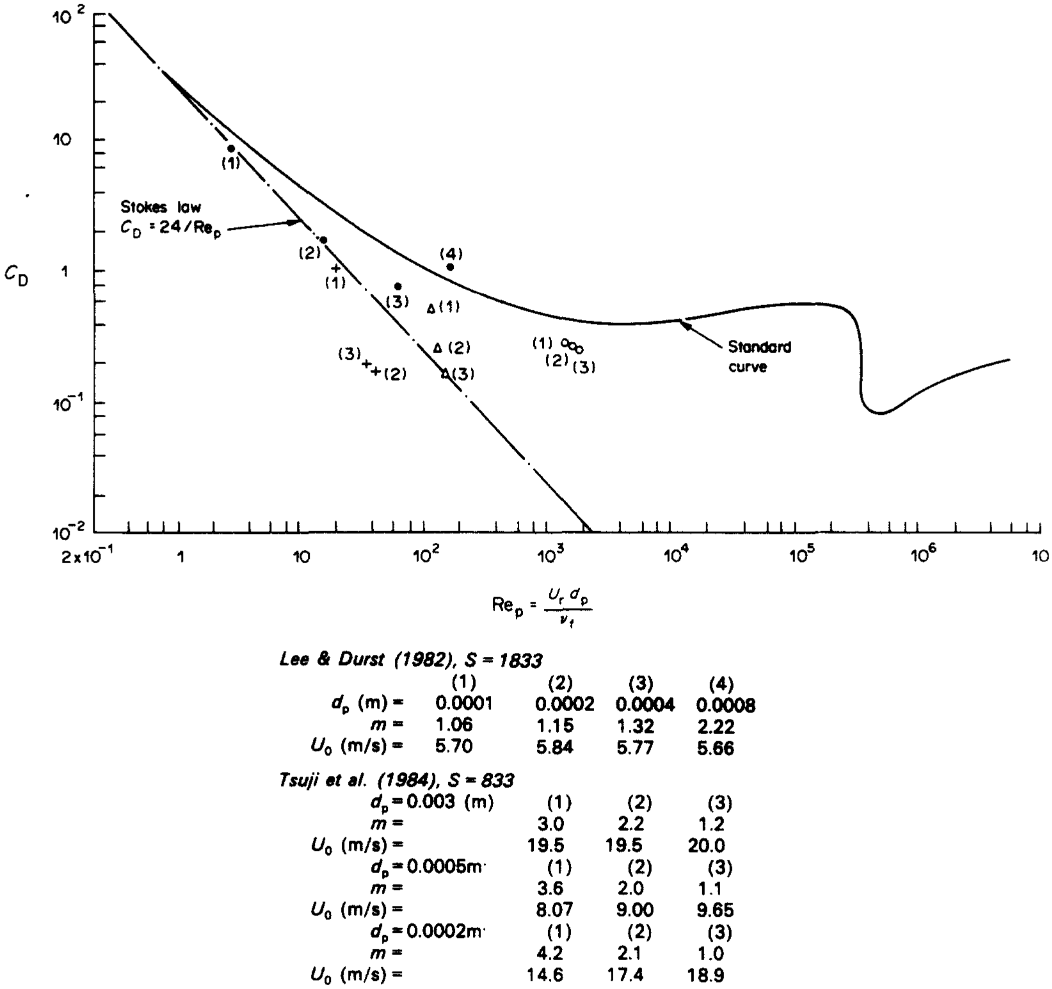


Figure 1. Drag coefficient for a particle in a suspension flow as a function of the particle Reynolds number based on the molecular viscosity of the fluid.

$$\frac{\mu_{fm}}{\mu_f} = \left(\frac{1-\alpha}{\alpha_m}\right)^{-2.5\,\alpha_m(\mu_p+0.4\,\mu_f)/\mu_p+\mu_f}, \qquad [3]$$

where

μ_{fm} = the modified viscosity of the fluids,

α_m = the maximum packing particle volumetric concentration ($\alpha_m = 0.62$ for solid particles)

and

μ_p = the viscosity of the particulate phase material ($\mu_p = \infty$ for solid particles by definition),

which becomes, for solid particles,

$$\frac{\mu_{fm}}{\mu_f} = \left(\frac{1-\alpha}{0.62}\right)^{-1.55}. \qquad [4]$$

Now, let us proceed to apply this proposed drag coefficient to the two abovementioned experiments (Lee & Durst 1982; Tsjui *et al.* 1984) on the local measurement of the flow properties of an upward solid particle–air two-phase turbulent suspension flow in a vertical pipe. The particle volumetric concentration α in these experiments varies between 0.6×10^{-3} and 8.0×10^{-3} and therefore the ratio μ_{fm}/μ_f computed from [4] will vary between 1.001 and 1.02. The computed modified particle Reynolds number will differ from the particle Reynolds number Re_p by a maximum of 2%. In figure

1, the experimental points based on the modified particle Reynolds number and the predicted drag coefficient C_D from this proposal is the standard drag curve. It is clear that the discrepancy between experimental results and predicted values from this proposal on the drag coefficient C_D is significant, mostly by an order of magnitude. Therefore, besides the particle volumetric concentration α, as considered in this proposal, there must be other factors which also play an important role in the establishment of the drag coefficient for particles in a turbulent two-phase suspension flow.

Torobin & Gauvin (1960) studied the effects of free-stream turbulence on the drag coefficient of fairly large spheres of different densities and diameters moving in steady motion in an upward cocurrent turbulent flow wind tunnel in which a novel arrangement of orifice grids in series was employed to create a flow with a flat central mean velocity profile and a random energy spectrum for the entire particle trajectory. It is important to note the difference between the turbulence intensities felt by a stationary sphere and by a moving sphere in the same turbulent flow. A stationary sphere feels the turbulence intensity of the flow relative to the velocity of the flow which is defined with respect to a stationary frame of reference. However, a moving sphere which is following the fluid feels the turbulence intensity of the flow relative to the slip velocity of the flow which is defined with respect to a frame of reference moving with the sphere. Therefore, a moving sphere feels a greatly enlarged turbulent intensity in the same turbulent flow. At low turbulent intensities, the values of the drag coefficient were found to coincide with those obtained in laminar fluids. At sufficiently high disturbances, there appeared a characteristic sharp drop in the drag coefficient of an order of magnitude to the level of around 0.1 at a value of the particle Reynolds number of somewhere between 10^2 and 10^4. This result generally agrees with the low level of the value of the drag coefficient of the present study in which the turbulence in the fluid is largely dominated by the arrays of particles in the suspension flow.

A graphic insight into what would occur at higher levels of free-stream turbulence has been provided by Ahlborn (1931) who qualitatively studied the effects of large disturbances on the boundary layer and wake of a fixed cylinder by a tracer photography technique. At a value of the Reynolds number of about 30, the flow around the cylinder with a laminar incident stream shows the usual early separation of the boundary layer which creates a sizable wake behind the cylinder. However, the flow for the same system at the same velocity but with a strip grid placed upstream to introduce what is obviously an intense free-stream turbulence shows a drastic delay in boundary layer separation. The separation point has been shifted far downstream and it appears to have the general separation characteristics of a turbulent boundary layer. Even more dramatic is the stunting action of the turbulence on the cylinder wake. The wake has essentially completely disappeared and the flow around the cylinder assumes the shape of a Stokes flow.

The Stokes law of drag for a sphere is derived on the assumption of very small particle Reynolds number ($Re_p \ll 1$). The small velocities associated with small particle Reynolds number, as shown in figure 2a, render the convection terms in the governing momentum equations negligible. Solutions for the flow field around the sphere become readily obtainable due to the linearization of these equations. The drag force computed from these solutions consists of two contributions, the viscous force distribution over the sphere surface and the pressure drag to the pressure force distribution over the sphere surface.

In the present case, in addition to contributing to the elimination of the wake behind the sphere, the high-intensity turbulence felt by a moving sphere has a dominant influence on the flow field around the sphere which can be conceptionally divided into a boundary layer along the surface of the sphere and an outer flow surrounding the sphere. The high-intensity turbulence most effectively brings high-momentum fluid to the boundary layer and thus reduces the outer, or higher-momentum, portion of the boundary layer. What is left of the boundary layer is a thin shear layer in which the convection of fluid momentum becomes insignificant, as shown in figure 2b. Thus it is expected that the viscous contribution to the drag on the sphere in the present case should be similar to that in the Stokes case with the molecular viscosity replaced by an apparent turbulent viscosity of the fluid as felt by the sphere.

In the outer flow, the high-intensity turbulence most effectively brings high-momentum fluid to the region adjacent to the outer surface of the reduced boundary layer. Thus the variation in fluid velocity in the outer flow becomes small and the convective transfer of fluid momentum becomes

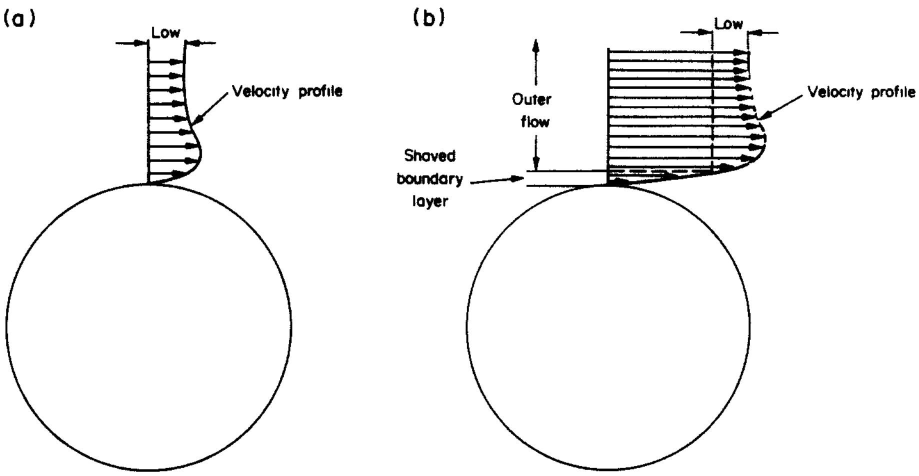


Figure 2a. Stokes flow around a sphere ($\mathrm{Re_p} \ll 1$).
Figure 2b. High-intensity turbulent flow around a moving sphere ($\mathrm{Re_p} > 1$).

insignificant, as shown in figure 2b. The pressure on the surface of the sphere is impressed through the thin shear boundary layer from the surrounding outer flow. Therefore it is also expected that the pressure contribution to the drag on the sphere in the present case should be similar to that in the Stokes case with the molecular viscosity replaced by an apparent turbulent viscosity of the fluid as felt by the sphere. And the total drag on the sphere in the present case should be similar to that in the Stokes case with a similarly replaced fluid viscosity.

A closer look at the plot in figure 1 reveals that the experimental points from widely varying flow conditions were found to be scattered around the extension of the Stokes law plot up to a value of the particle Reynolds number of $\mathrm{Re_p} = 2000$. It becomes clear that the use of the particle Reynolds number $\mathrm{Re_p} = U_r d_p/\nu_f$ based on the molecular kinematic viscosity of the fluid used in this plot is incorrect and an equivalent turbulent kinematic viscosity of the fluid for the particles in a suspension flow should be used instead. This apparent turbulent kinematic viscosity can be obtained from the Stokes law from the experimental values of the drag coefficient C_D and is expected to be a function of the particle size and concentration in the suspension flow. The particle volumetric concentration in the suspension can be computed from the particle-to-air mass flux ratio m,

$$\alpha = \frac{1}{\dfrac{S}{m}\dfrac{U_p}{U_0} + 1} \qquad [5]$$

where α = particle volumetric concentration. The particle size d_p can be made non-dimensional by the introduction of the Froude number

$$\mathrm{Fr} = \frac{U_0}{(d_p g)^{0.5}}. \qquad [6]$$

The following correlation for the apparent turbulent kinematic viscosity of the fluid for the particles in a suspension flow as a function of the particle volumetric concentration α and the Froude number Fr, the flow Reynolds number Re and the particle-to-air density ratio S is obtained:

$$\frac{\tilde{\nu}_f}{\nu_f} = 100\,\alpha^{0.5}\,\mathrm{Fr}^{-2.33}\,\mathrm{Re}^{0.86}\,S^{0.3} \quad \text{for} \quad \tilde{\mathrm{R}}\mathrm{e_p} > 10$$

$$\frac{\tilde{\nu}_f}{\nu_f} = 1 \quad \text{for} \quad \tilde{\mathrm{R}}\mathrm{e_p} < 10, \qquad [7]$$

for

$$0.6 \times 10^{-3} < \alpha < 8.0 \times 10^{-3},$$

$$50 < \mathrm{Fr} < 333,$$

$$1.2 \times 10^4 < \mathrm{Re} < 3.1 \times 10^4,$$

$$800 < S < 1900$$

and

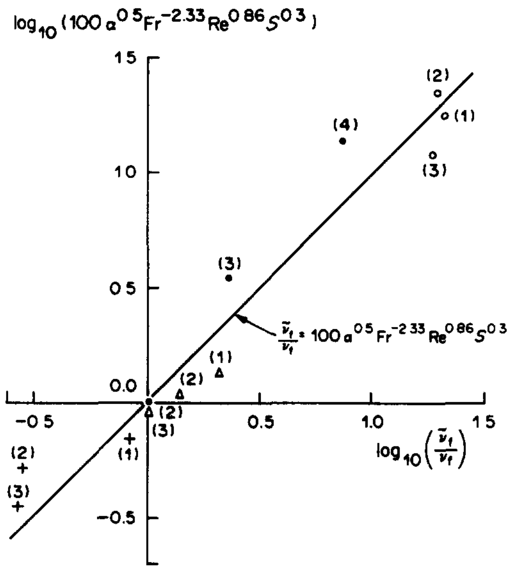
$$\mathrm{Re}_p < 1800,$$

where

$\tilde{\nu}_f$ = the apparent turbulent kinematic viscosity of the fluid for the particles in a suspension flow.

Figure 3 shows a comparison of this correlation with experimentally determined points. The apparent turbulent kinematic viscosity of the fluid for the particles can be larger or smaller than the molecular kinematic viscosity of the fluid depending on the particle size and concentration. In general, for large particles and large concentrations, the apparent turbulent kinematic viscosity of the fluid for the particles is larger than the molecular kinematic viscosity of the fluid. And, for small particles and small concentrations, the reverse is true.

Figure 4 shows a plot of the particle drag coefficient against a turbulent particle Reynolds number $\tilde{\mathrm{Re}}_p = U_r d_p/\tilde{\nu}_f$ based on the apparent turbulent kinematic viscosity of the fluid $\tilde{\nu}_f$ for the particles in a suspension flow. The experimental points are found to cluster closely around the Stokes law of drag extended to large values of the particle Reynolds number Re_p.



Lee & Durst (1982), S = 1833

	(1)	(2)	(3)	(4)
d_p (m) =	0.0001	0.0002	0.0004	0.0008
m =	1.06	1.16	1.32	2.22
U_0 (m/s) =	5.70	5.84	5.77	5.66

Tsuji et al. (1984), S = 833

d_p = 0.003m	(1)	(2)	(3)
m =	3.0	2.2	1.2
U_0 (m/s) =	19.5	19.5	20.0
d_p = 0.0005m:	(1)	(2)	(3)
m =	3.6	2.0	1.1
U_0 (m/s) =	8.07	9.00	9.65
d_p = 0.0002m:	(1)	(2)	(3)
m =	4.2	2.1	1.0
U_0 (m/s) =	14.6	17.4	18.9

Figure 3. Correlation of the apparent turbulent kinematic viscosity of the fluid for particles in a suspension flow.

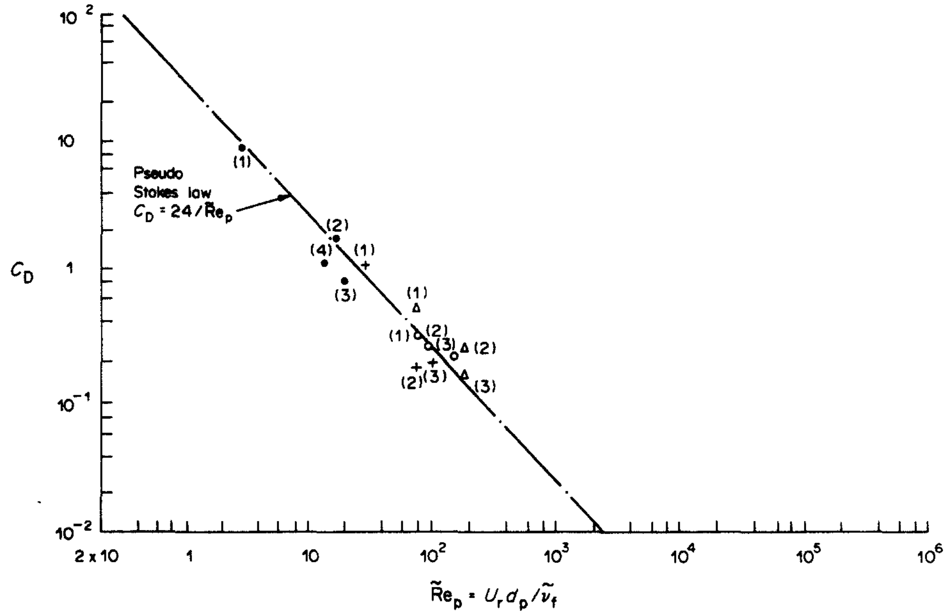


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Figure 4. Drag coefficient for a particle in a suspension flow as a function of the particle Reynolds number based on the apparent turbulent kinematic viscosity of the fluid for particles in a suspension flow.

The correlation of $\tilde{\nu}_f/\nu_f$ of [7] is derived for flow conditions along the centerline of the pipe and can be readily extended to other radial positions in the pipe. Its dependence on the flow Reynolds number reflects the influence of the background turbulence inherent in the pipe flow. From the single-phase flow measurements of Laufer (1953), Lee & Durst (1982) and Tsuji *et al.* (1984), the following correlation for the longitudinal velocity fluctuation u' along the centerline of the pipe can be formulated:

$$\left(\frac{D}{\nu_f}u'\right) = 0.2\,\mathrm{Re}^{0.86}. \qquad [8]$$

Substituting the Reynolds number Re from [8] into [7] and identifying u' with the local single-phase flow longitudinal velocity fluctuation at any radial location, we have the following correlation for any radial location:

$$\frac{\tilde{\nu}_f}{\nu_f} = 500\,\alpha^{0.5}\,\mathrm{Fr}^{-2.33}\,\mathrm{Re}'\,S^{0.3} \quad \text{for} \quad \tilde{\mathrm{Re}}_p > 10$$
$$\frac{\tilde{\nu}_f}{\nu_f} = \frac{u'}{(u')_c} \quad \text{for} \quad \tilde{\mathrm{Re}}_p < 10, \qquad [9]$$

where $\mathrm{Re}' = Du'/\nu_f$, the local flow turbulence Reynolds number and $(u')_c$ is u' for the pipe center, for the range of the flow parameters of correlation of [7].

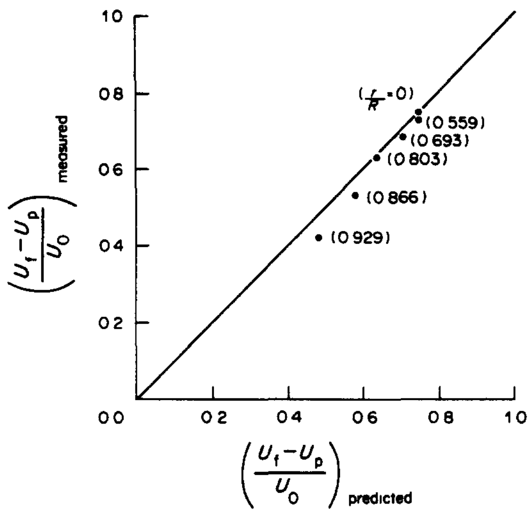

Figure 5. Comparison of the measured slip velocity against predicted values based on the drag correlation across the pipe radius [Lee & Durst (1982), $d_p = 800\ \mu$m].

Figure 5 shows a comparison of measured slip velocity across the pipe radius for the case of $d_p = 800\ \mu$m of Lee & Durst (1982) against predicted values based on the drag from the correlation of $\tilde{v}_t/v_f$ of [9] using the measured u' (assuming small variation in α).

DISCUSSION

From the reasoning outlined previously, the drag on a particle in a suspension flow is expected to be governed by the Stokes law of drag modified to be in terms of a turbulent particle Reynolds number. For large particles, as expected, the apparent turbulent kinematic viscosity of the turbulent Reynolds number has been found to be greater than the molecular kinematic viscosity of the fluid. For small particles, however, the reverse is true and a qualitative explanation can be formulated from an analysis of a particle's dynamic response to the transverse fluctuations of the fluid motion.

Lee & Wiesler (1987) and Lee (1987) have developed a theoretical model to explain the behavior of transverse particle transport in a turbulent two-phase suspension flow. This model is based on the ability of a particle to respond to the surrounding fluid motion which consists of three separate components representing, respectively, the mean motion, the turbulent fluctuations and an apparent drifting due to the effect on the oscillatory component of the fluid motion by the concentration distribution of particles. In general, a large and heavy particle would respond mainly to the mean fluid motion and the drag it experiences in the longitudinal direction is related to its velocity relative to the mean fluid motion. On the other hand, a small and light particle would respond more to the fluid fluctuations than to the mean fluid motion in the transverse direction. Such a particle therefore would become relatively loose in its transverse position on exposure to the fluid fluctuations in the longitudinal direction. It is conceivable that, instead of responding to the mean fluid motion in the longitudinal direction, the particle would tend to respond to the most significant downward component of fluid fluctuations in its immediate neighborhood. A Stokes law description of the drag based on the mean motion of the fluid and the particles in the longitudinal direction can thus produce, for small particles in a suspension flow, an apparent turbulent kinematic viscosity smaller than the molecular kinematic viscosity of the fluid.

CONCLUSION

Based on an analysis of the results of local measurements of solid particle–air two-phase turbulent suspension flow in a vertical pipe, the following conclusions are drawn:

1. The important dynamic interaction between the phases in still governed by the simple Stokes law of drag extended to large values of a turbulent particle Reynolds number in which, instead of the molecular viscosity, an equivalent turbulent viscosity of the fluid for the particles in the suspension flow is used.
2. A correlation has been found for this apparent turbulent viscosity of the fluid for the particles in the suspension flow in terms of particle size and concentration, the local flow turbulence Reynolds number and the particle-to-fluid density ratio.

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